### 3.6.2 The Multiple Scattering Algorithm

The multiple scattering algorithm will compute both the multiple scattering factor ($\eta$) used to correct the optical depth and extinction retrievals as discussed in the above section, and the range-to-surface delay. The latter is not used by the atmospheric processing routines, but is output on the GLA11 product for use by the altimetry processing group. The multiple scattering algorithm does very little in the way of computation. It merely obtains values of $\eta$ and range-to-surface delay from pre-calculated lookup tables based on the values of the inputs described below.

The multiple scattering factor $\eta$ depends on the extent to which photons in the pulse have their trajectories altered by scattering events. This in turn is a function of the microphysical and physical properties of the cloud and aerosol layers in which the emitted photons are scattered. Specifically, the degree of scattering depends on (a) particle sizes within a scattering layer, (b) the layer optical depth, (c) the proximity of the scattering layer to the surface (for the range-to-surface delay), and (d) the physical thickness of the layer. It is important to understand that each of these factors is examined here independently, and the actual multiple scattering factor and scattering-induced range-to-surface delay are a result of both competing and additive effects from these various sources.

The following inputs are used in the algorithm

     a.  Layer type
     b.  Layer Top Height
     c.  Layer Bottom Height
     d.  Layer Average Temperature
     e.  Estimated Effective Optical Depth of the Layer
     f.  Latitude of Observation
     g.  Longitude of observation

As mentioned above, these inputs are used to index into pre-calculated tables that contain corresponding values of the multiple scattering correction factor and the range-to-surface delay. The tables are explained in more detail in section 3.6.2.1 below, but first we examine the major inputs to the multiple scattering algorithm and note their effects on the multiple-scattering factor and the range-to-surface delay. In each instance, other sources are neglected; e.g., when optical depth is examined, variations in layer thickness, particle size, etc. are not considered. It must therefore be borne in mind that the relationships outlined here will likely differ from those seen in observations of scattering layers.

1. *Layer optical depth*

This quantity is directly input to the algorithm. Increases in the optical depth of the scattering layer make more surfaces available for scattering, thereby increasing the potential for photons to be removed from the emitted direction. All other things being equal, therefore, optical thick layers (both clouds and aerosols) will scatter away more photons that thin layers. The layer optical depth is important for both $\eta$ and the range-to-surface delay.

2. *Proximity of the scattering layer to the surface*

This input quantity is obtained from the bottom of the layer, and from knowledge of the surface elevation obtained from a pre-determined lookup table (DEM) over all longitudes and latitudes. It

is important for the calculation of the range-to-surface delay. It is not a factor in the computation of $\eta$.

When a photon is scattered high in the atmosphere, it travels further along the scattered direction prior to reaching the surface, than it would if the scattering event occurred nearer to the surface. Very near the surface, most scattered photons, regardless of the angle at which they are scattered away from the emitted direction, will likely remain within the view of the receiver. Higher in the atmosphere, the range of scattering angles at which photons will still remain within the field of view is more limited, and only those scattering events that occur at small enough angles to retain the photon within the instrument's field of view contribute to the magnitude of the multiple scattering factor and range-to-surface delay.

3. *Layer physical thickness*

This input quantity is obtained from the layer bottom and layer top heights. The geometric thickness of the scattering layer is important for both the range-to-surface delay and $\eta$.

The physical thickness of the cloud impacts scattering-induced delay in much the same way that the height of the cloud above the surface does. As we saw above, all other things being equal, scattering events closer to the surface are more likely to contribute to the scattering factor. This understanding can be extended to the physical thickness of the cloud as follows. If all scattering surfaces are concentrated in a narrow layer at the base of the layer, i.e. if the cloud (or aerosol) is physically thin, this is equivalent to stating that the scattering occurs nearer the surface. If, on the other hand, the scattering surfaces are distributed over a large physical thickness of the cloud, some scattering events take place near the surface, whereas others occur at larger distances from it. Scattering events from the higher elevations within the layer are likely to direct the photons away from the field of view of the receiver, and cause little multiple scattering effects recorded by the receiver.

4. *Cloud particle effective radius*

This input quantity is obtained from a look-up table computed as a function of latitude, longitude, layer height and temperature using information from satellite studies of clouds and aerosols across the globe. Figure 3.6.1 shows how the diffraction peak of scattering varies with particle size; along with oscillating values for mono-dispersed spheres, the curve for a broad distribution of particle sizes is also shown. As particle sizes become larger, a decreasing fraction of the emitted energy is forward scattered; this reduces the number of photons that remain within the footprint of the receiving instrument. At some threshold value of particle radius, around 15-20 microns, this relationship is bounded, and a fixed fraction of the emitted energy then remains within the forward scattered portion, and further increases in particle size do not cause any changes. Thus, both the atmospheric multiple scattering effect ($\eta$) and the range-to-surface delay depend on the particle size.

The I-SIPS software will retrieve particle size from this lookup table, which will be generated and supplied by the science team. Here we define the size of the table and how the particle size will be obtained from the table. There will be 18 latitudes, 36 longitudes, 20 temperatures and 8 heights for a total of 103,680 entries. Thus, the particle size lookup table will be a 4 dimensional array as: P (18,36,20,8). Accessing the appropriate bin will be a matter of computing the necessary indices from the spacecraft position (latitude/longitude) and the layer height and associated temperature (from the MET data). If I, J, K and L are the four indices of the particle size array, then:

I = (latitude + 90) / 10 + 1
J = longitude / 10 + 1
K = (T − 180)  * 20 / 130 , where T is the layer temperature in Kelvin.
And L is calculated as shown in table 3.6.1.

**Table 3.6.1.** Calculation of the 4th index of the
particle radius lookup table as a function of layer height.

| L | Layer Height (km) |
|---|---|
| 1 | < 0.200 |
| 2 | 0.200 – 0.500 |
| 3 | 0.500 – 1 |
| 4 | 1 – 2 |
| 5 | 2 – 4 |
| 6 | 4 – 8 |
| 7 | 8 - 12 |
| 8 | > 12 |

*3.6.2.1 Multiple scattering factor (η) and Range delay tables*

Monte Carlo calculations will be made by the science team to generate tables of the multiple scattering correction factor (η) and the range-to-surface delay. The details of how these tables are generated are not presented here. Of concern to the I-SIPS software is only the generation of the proper indices (based on the inputs) from which to retrieve the information from the tables. The multiple scattering correction factor will be a function of optical depth, particle size and the geometric depth of the scattering layer. Much like the particle size table described above, these tables will be accessed by computing the array index from the appropriate input parameter. Specifically, the multiple scattering factor table will have three dimensions: $\eta(\tau,r,d)$, where $\tau$ is the layer optical depth, r is the particle size, and d is the geometric depth of the layer. The dimension of this table will be (12,11,7). If I, J, and K are the 3 indices of the array, their values are defined from the inputs as shown in tables 3.6.2, 3.6.3 and 3.6.4.

**Table 3.6.2.** Calculation of the first index of the
multiple scattering correction table as a function of
layer effective optical depth.

| I | Layer Optical Depth |
|---|---|
| 1 | < 0.02 |
| 2 | 0.02 – 0.05 |
| 3 | 0.05 – 0.10 |
| 4 | 0.10 – 0.20 |
| 5 | 0.20 – 0.35 |
| 6 | 0.35 – 0.60 |
| 7 | 0.60 – 0.90 |
| 8 | 0.90 – 1.20 |
| 9 | 1.20 – 1.60 |
| 10 | 1.60 – 2.00 |
| 11 | 2.00 – 2.50 |
| 12 | > 2.50 |

**Table 3.6.3.** Calculation of the second index of the multiple scattering correction table as a function of input particle size.

| J | Particle Size (μm) |
|---|---|
| 1 | < 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 5 |
| 5 | 7 |
| 6 | 10 |
| 7 | 14 |
| 8 | 20 |
| 9 | 30 |
| 10 | 50 |
| 11 | > 50 |

The particle sizes that are retrieved from the particle size table defined above can take on any value. The Monte Carlo calculations will be performed only for the specific particle sizes listed in table 3.6.3. Generally, the particle size will fall somewhere between the listed values and we will linearly interpolate to obtain the actual value of $\eta$ or the range-to-surface delay. As an example, let us suppose that the particle size retrieved from the lookup table is 4.7 μm. To obtain a value of $\eta$, we first locate the particle size value in table 3.6.3 that is greater than 4.7μm. Call this index J2 (which in this case = 4). J1 is defined as J1 = J2 − 1. Suppose further, that the value of $\eta$ retrieved from the multiple scattering lookup table for J1 = 3 is 0.5 and J2 = 4 is 0.6. Then the value of $\eta$ for 4.7μm is calculated as:  0.5 + (0.6 - 0.5)(4.7 - 3.0)/(5.0 - 3.0), which equals 0.585. The generalized equation for the calculation of $\eta$ for a given particle size (r) is:

$\eta = \eta(J1) + [\eta(J2) − \eta(J1)] (r − r1) / (r2 − r1),$

where r1 and r2 are the particle sizes corresponding to J1 and J2.

It should be noted that for particle sizes less than 1μm, that the value of $\eta$ at J=1 is used, and that for particle sizes greater than 50μm, the value of $\eta$ at J=11 is used. Also note that interpolation is performed only for the particle size dependence. The other variables upon which $\eta$ depends (layer optical and geometric depth) are not interpolated. In other words, the values retrieved from the $\eta$ table for optical depth values within a bin (e.g. 0.023, 0.035, etc)  will be identical for a given particle size.

**Table 3.6.4.** Calculation of the third index of the multiple scattering correction table as a function of input layer geometric depth.

| K | Layer Geometric Depth (km) |
|---|---|
| 1 | < 0.200 |
| 2 | 0.200 – 0.500 |
| 3 | 0.500 – 1.0 |
| 4 | 1.0 – 2.0 |
| 5 | 2.0 – 3.5 |
| 6 | 3.5 – 6.0 |
| 7 | > 6.0 |

Once all the indices are calculated, the multiple scattering correction factor ($\eta$) is retrieved from the table. The multiple scattering warning flag (F) is calculated from the value of $\eta$ using the following equation:  F =  IFIX ((1.0 - $\eta$) * 14). The multiple scattering warning flag is output to the GLA11 product.

The range-to-surface delay (in nanoseconds) is a function of the layer effective optical depth, particle size, layer geometric depth and the height of the layer above the surface. Thus, it will be a four dimensional array with dimensions: (I=12,J=11,K=7,L=13). The I, J, and K indices are calculated from tables 3.6.2, 3.6.3 and 3.6.4, respectively. The last index (L) is calculated from table 3.6.5, which shows the index value as a function of the height of the bottom of the scattering layer above the local surface. Note that this must be calculated by subtracting the DEM value for the given latitude/longitude from the retrieved layer bottom height.

**Table 3.6.5.** Calculation of the fourth index of the range-to-surface delay table as a function of layer bottom height.

| L | Layer Bottom Height (km) |
|---|---|
| 1 | < 0.050 |
| 2 | 0.050 – 0.100 |
| 3 | 0.100 – 0.200 |
| 4 | 0.200 – 0.300 |
| 5 | 0.300 – 0.500 |
| 6 | 0.500 – 0.700 |
| 7 | 0.700 – 1.0 |
| 8 | 1.0 – 1.4 |
| 9 | 1.4 – 2.0 |
| 10 | 2.0 – 2.5 |
| 11 | 2.5 – 3.0 |
| 12 | 3.0 – 3.5 |
| 13 | > 3.5 |

In summary, the algorithm will produce following quantities which will be written out to the GLA11 data product:

a. Multiple scattering factor (ranges from 0 to 1)
b. Surface range delay estimate (nanoseconds)
c. Multiple Scattering Effect Warning Flag (ranges from 0 to 15)
d. Particle sizes estimated and used in the scattering calculation